INTRODUCTION

The three major political groups that vote in presidential elections in the U.S. are Democrat, Republican, and Independent. When it comes time, voters will take into account the past record of the president and use that information to choose whom to vote for in the next presidential election. Suppose that of the registered voters in the U.S., 80% of the Democrats will vote Democratic again, 15% of the voting Democrats will switch and vote Republican, and 5% of voting Democrats will switch and vote Independent. Of those who voted Republican in the last election, 30% will vote Democratic, 60% will vote Republican again, and 10% will vote Independent. Of the Independents, 25% will vote Democratic, 25% will vote Republican, and 50% will vote Independent again.

Making sense of this information is difficult to grasp all at once; however, if we organize it into a display like the one below, we begin to see some advantages that such a display has over the verbal description above.

\[
T = \begin{pmatrix}
    0.80 & 0.15 & 0.05 \\
    0.30 & 0.60 & 0.10 \\
    0.25 & 0.25 & 0.50
\end{pmatrix}
\]

The table of numbers \( T \) shown above is an example of a matrix in which the numbers are known as entries. The dimension of a matrix is given by the number of rows (always given first) followed by the number of columns (given second). For example, the matrix \( T \) above is considered a 3 \( \times \) 3 matrix. If a matrix has \( m \) rows and \( n \) columns, then it is said to be an \( m \times n \) matrix. If \( m = n \), as in \( T \), then the matrix is said to be a square matrix.

Row number and column number, in that order, identify individual entries in a matrix. For example, the number 0.10 is the entry in row 2 and column 3 of \( T \), abbreviated as \( T_{23} \). We also have \( T_{32} = 0.25 \) and \( T_{13} = 0.05 \) as two other entries in the matrix \( T \).

Each entry in \( T \) has a specific, unique meaning; therefore, the dimension of a matrix cannot be reduced without losing essential information. In \( T \), the rows represent the political party of the candidate people voted for in the last presidential election, whereas the columns represent the party of the candidate that the voters will choose to vote for in the next election. \( T \) is an example of a transition matrix, for it contains information concerning the voter’s transition from a present political party to the party of a candidate in the next election. The concept of a transition matrix is essential to the later section on Markov chains.
As we have seen in the previous sections, mathematical modeling can help us calculate, simulate, measure, model, predict, interpret, and do much more to describe the behavior of real world situations, especially when the phenomena being measured is approximated by a continuous function.

A matrix, on the other hand, can be used with a collection of data that does not lend itself to the use of a continuous model. Matrices (the plural of matrix) are important in a larger branch of mathematics called discrete mathematics. The term discrete refers to the fact that the techniques deal with finite sets of non-continuous data. The advantage of using a matrix to organize a set of data can be seen in the preceding example of changing voting habits.

The function $f(x) = x^2$ is a continuous model for certain real-world phenomena (the trajectory of an object tossed into the air, for example). On the other hand, a matrix is a mathematical tool that handles data sets in summary form. Because matrices operate with discrete data, they possess dimension; that is, a matrix cannot be reduced to a single number such as the value of a function at a point. A matrix can be thought of as a single entity for the sake of simplicity; however, it is a single entity that contains many data entries. Furthermore, just as functions adhere to certain algebraic rules and operations, matrices have a special algebra associated with them. The following sections will introduce you to methods for using matrices to model discrete data.